Main factors of climate variability and their application for environment protection problems in Siberia

Vladimir Penenko & Elena Tsvetova

Institute of Computational Mathematics and Mathematical Geophysics SD RAS
Novosibirsk
Algorithms
for revealing climatic variability

- Singular vectors (SV) for forward tangent operator of dynamical models and the use of SV-decomposition for scenario construction and errors analysis (uncertainty reducing);
- ensembles of prognostic scenarios with generation of perturbations (“breeding cycle”, Lyapunov’s vectors);
- Monte-Carlo methods for scenario construction
- Stochastic-dynamic moment equations and Liouville equations
  ICMMG technology
- Orthogonal decomposition of the phase spaces of non-linear dynamical systems for formation of informative basis subspaces;
- Minimization of uncertainties with respect to given criteria of prognosis quality (+ data assimilation if any)
Scenarios construction
and adaptive monitoring with SV

\[ \frac{\partial \phi}{\partial t} + A(\phi) = 0 \Rightarrow \]
Tangent linearization about \( \tilde{\phi}(x,t) \)

\[ \frac{\partial \delta \phi}{\partial t} + A_L \delta \phi = 0 \quad \delta \phi(x,0) = (a\ priori) \]

\( \delta \phi(x,t) = L \delta \phi(x,0), \tilde{x} \in D, t \in [0, \bar{t}] \)

\( L(x,t) \) - forward tangent propagator about \( \tilde{\phi}(x,t) \)

\[ \psi(x,0)|_D = L^* [\delta \phi(x,t)]_{\Sigma_t}, t \in [\bar{t} \rightarrow 0] \]
Basic relations and patterns for SVs

\[ \|\delta \varphi(t)\|_{\Sigma_t} = (\delta \varphi(t), \delta \varphi(t)) = \]
\[ = (L \delta \varphi(0), L \delta \varphi(0)) = (\delta \varphi(0), L^* L \delta \varphi(0)) = \]
\[ = (\delta \varphi(0), \psi(0)) \]

\[ \sum_{t \in D} \text{evaluation domain at } t = \bar{t} \]
\[ \sum_{0 \in D} \text{target area at } t=0 \]
\[ [0, \bar{t}] \text{ “optimal” time interval } (\leq 48 \text{ h}) \]
Partial eigenproblem for SVs

\[ L^*LV_i = \sigma_i^2 V_i, \quad (i \in K) \]

\( \sigma_i, V_i \) singular values and vectors of L (SEVs, SVs)

- Lanzosh algorithm
- Orthogonal decomposition of perturbation spaces
- Optimal construction of perturbations with respect to rapidly growing SVs
Structuring and decomposition of data bases

Initial data base \[ \Phi \equiv \{ \varphi(\tilde{x},t,\tilde{Y}) \in Q(D_t) \subset R_N, \tilde{Y} \in R(D_t) \} \]

Structured data base \[ Z = \{ z_i = C^{1/2} \varphi_i, \ i = 1, n, \ \varphi_i \in R_N \} \]

\[ Z \quad n \times N \] matrix of vectors from \( R_n \times R_N \)

\[ C \quad N \times N \] diagonal matrix of total energy weight of \( \varphi \)

Scattering function \[ S(v) = (v^T Z^T Z v) = (v^T \Gamma v) \]
Orthogonal decomposition of \( Z \) on the base of optimal properties of \( S(\nu) \)

\[
\begin{align*}
\Gamma \nu = \lambda \nu &\Rightarrow \{ \lambda_p, \nu_p \in R_n \}, \quad \Psi_p \in R_N \\
\nu_p^T \nu_q = \lambda_p \delta_{pq}, \quad \Psi_p^T \Psi_q = \delta_{pq}, \quad p, q = 1, n
\end{align*}
\]

\[
\begin{aligned}
V &= \{ \nu_p \} \\
\Lambda &= \text{diag}\{ \lambda_p > 0 \} \\
\Psi &= \{ \Psi_p \in R_N, \quad p = 1, n \}, \quad n \times N \text{ matrix}
\end{aligned}
\]

\[\Psi \equiv Z \Lambda^{-1} \quad \text{decomposition algorithm}\]

\[\tilde{Z} = \Psi V^T \quad \text{reconstruction algorithm}\]
Factor subspaces
for deterministic- stochastic scenarios

• Factor spaces

\[ \mathbf{r} = \mathbf{X}_0 + \mathbf{x} \]

is a linear subset of the vector space \( X \) \( \mathbf{DATA} \)

\( \mathbf{x} \) is arbitrary element from \( X \)

!! Algebraic operations in \( X \) leave \( \mathbf{X}_0 \) invariant
\( \mathbf{X}_0 \) is the leading phase space,
\( \mathbf{x} \) are generated perturbations
Construction of the vector set \( X_0 \)

\[
X_0 = \sum_{i=1}^{n_d} c_i Y_i, \quad n_d \leq n, \quad 0 \leq c_i \leq \max |s| \quad \forall j
\]

Formation of vectors \( X \)

1. Deterministic case:
   calculation by means of the process models
2. Deterministic-stochastic case:
   \( c_i \) generation by means of the stochastic processes
   of the fractal type described by gaussian process with variance
   \[
   \sigma_q^2 = \lambda_q^{2H}, \quad 0 \leq H \leq 1
   \]
   \( H \) is a parameter of the fractal size,
   \( l_q \) are the eigenvalues of the Gram matrix
Forming the guiding phase space with allowance for observation data on the subdomain

\[ Z^m(x, \tau) \] measured data; \( \Psi_p(x, t) \) basis

\[ Z(x, t) = \sum_{p=1}^{n_a} a^m_p \Psi_p(x, t), \ (x, t) \in D_t, \ n_a \leq n \]

\[
\min_{\{a^m_p\}} \left\| Z^m(x, \tau) - \sum_{p=1}^{n_a} a^m_p \Psi_p(x, \tau) \right\|^2_{D^m_{\tau}} \]

\[ \mathbf{a} = \left( \Gamma^m \right)^{-1} \mathbf{F}^m \]

\[ \mathbf{a} = \{ a^m_p, \ p = 1, \ldots, n_a \} \]

\[ \Gamma^m = \left\{ \Gamma^m_{pq} = \left( \Psi_p, W^m \Psi_q \right)_{D^m_{\tau}}, \ p, q = 1, \ldots, n_a \right\} \]

\[ \mathbf{F}^m = \sum_{p=1}^{n_a} \left( \Psi_p, W^m Z^m \right)_{D^m_{\tau}} \]

If \( \tau < t \) then \( Z(x, t) \) is forecast!
Winter pattern of the global 500-hPa geopotential height (the 1st main factor) 1950-2005

January 15

Level value:
-0.85 -0.71 -0.56 -0.42 -0.28 -0.13 0.01 0.16 0.30 0.45 0.59
Winter pattern of global circulation (the 1st main factor) 1950-2005
Summer pattern of the global 500-hPa geopotential height (the 1st main factor) 1950-2005

July 15
Summer pattern of global circulation (the 1st main factor) 1950-2005
East Siberia Region

90-140 E, 45-65 N

June 1950-2005
Variability of the phase spaces with respect to the first main factors

Eigenvectors N1, June, 1950-2005

Global, 17%

Eastern Siberia, 17,9%
Quantification of subspace scales

Eigenvalues of Gram matrices, June, 500-hPa, 1950-2005

Global scale

Regional scale
Monthly risk functions for Lake Baikal

July

December
Conclusion

• The set of numerical algorithms for orthogonal decomposition of the phase spaces of dynamical system evolution is developed for climate and ecology studies.

• The methods are applied for construction of long-term scenarios for risk assessment with respect to anthropogenic impact.

• This allows us to take into account climatic data for environmental studies of global and regional scale.
Acknowledgements

The work is supported by
• RFBR
  Grant 07-05-00673
• Presidium of the Russian Academy of Sciences
  Program 16
• Department of Mathematical Science of RAS
  Program 1.3.